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THE MINORITY QUESTION A GAME THEORY AND THEORETICAL ECONOMIC APPROACH

Introduction

The goal of this study is to present an approach with the help of which the minority problems are easily analyzable and understandable. This approach is based on the concepts and methods of the game theory and of the theoretical economics.

The majority - minority relation is considered in the context of the situation in which on the one hand the majority is the politically dominant group, and on the other hand the two communities have strong economic relations. The consequences of the first assumption are that the political decisions of the government reflect mainly the political aspirations of the majority. The second assumption means that the economic relations between the members of the two communities are numerous and are not regulated in the extent as in the case of the citizens of different states.

The classification of the majority / minority relations used in the article is an abstract classification. The data used in the examples are abstract data, selected in order to sustain the theoretical discussion behind of them.

More detailed discussion of the theoretical models and results used in the article can be found in [RASMUSSEN90], [STEFANESCU81] - game theory -, and [VARIAN91], [SAMUELSON92] - microeconomic theory.

Classification of the Majority / Minority Relations

The majority / minority relations can be classified on the base of the direction of the existing aspirations of assimilation.

Let's consider ***A* type** relation, a relation type in which the minority wishes to assimilate to the majority but the majority rejects the minority. An example for this type of relationship could be the Romanian - Roma relation in Romania, or the white American - black American relation (mainly until the 50s). One of the most important question in this kind of relationship which determines its nature is if there is or there isn't an objective discrimination criteria (ex. the color of the skin), based on which an objective distinction can be made between the members of the two communities. When the discrimination is based mainly on subjective criteria (the majoritarians can't accept the culture, the behavior of the minoritarians, or they think that the minoritarians are inferior to them, then the assimilation is more unacceptable for the minoritarians than in the case when there is an objective distinction criteria.

Lets consider ***B* type** relation, a relationship, where the majoritarians wish to assimilate the minority, but the minority oppose to this. An example for this type could be the Romanian - Hungarian relation from Romania. In such cases the minoritarians prefer discrimination (of course positive discrimination, at least on the subjective level), because they think they are superior to the majoritarians. In such a case, in a short run, even an objectively negative discrimination can be interpreted by the minoritarians as a positive thing, which proves that the majoritarians need primitive methods to rule over the minoritarians, otherwise they fear that the minoritarians will culturally dominate the majoritarians.

A Game Theory Interpretation of the Majority / Minority Relation

The game theory approach is applied to a simplified situation (similar descriptions can be find in [ELSTER95], p.30 - 37., or in [VARIAN91], p.556 - 570). In this situation each partner has two acting possibilities. The two extremes possibilities are: assimilating and rejecting the partner which two possibilities will be symbolized by **Rejection** (R) and **Assimilation** (A). In the case of the minority partner which wishes to assimilate, we can establish two possibilities seen from the point of view of the majoritarians: the **Nonconform** (N) behavior and the **Conform** (C) behavior. The majority which wishes to assimilate the minority can be theoretically for **anti - Minoritarian** (M) legislation and for **Pro - minoritarian** (P) legislation. In the presented models the values of the pay-off matrix are chosen in order to illustrate the possible situations, expressed by the gains or losses of each partner.

A type relation:

Majoritarian / Minoritarian	**Reject** / R	**Assimilate** / A
Nonconform / N	1, 1	3, -1
Conform / C	-1, 3	5, 5

Tabl. 1.

If we make an abstraction from the actual values, the meaning of the values of the pay-off matrix, is the following:

N, R: If the majority is rejecting the minority, and the minority has a nonconformist behavior, both communities are gaining something. On the one hand the majoritarians are conserving the subjective pureness of their community, on the other hand the minoritarians can live in such conditions, in which the costs of the emigration (cumulative subjective and objective costs) are higher than the costs of the remaining.

N, A: If the majority is accepting the minority, but the minoritarians have nonconformist behavior, the minority gains more than in the previous case, and the majority looses subjectively, and possibly objectively too. The minoritarians are enjoying the benefits of the accepting behavior of the majority, of the pro-minoritarian legislation, without modificating, or conforming their behavior. The majoritarians are resigning from their subjective pureness, and possibly are discriminating positively the minoritarians too, without any subjective or objective benefit.

C, R: If the majority is rejecting the minority, and the minoritarians try to conform their behavior to the majoritarian claims, the majoritarians are gaining benefits, and the minoritarians are loosing a lot. The majoritarians are conserving their subjective pureness, and have less trouble with the minoritarians' nonconformist behavior. The minoritarians are giving up their customs, are loosing subjectively, and it is possible that they will loose objectively, too.

C, A: If the majoritarians have an accepting behavior, and the minoritarians are conforming their behavior to the majoritarian claims, the majoritarians will have less trouble with nonconformist minoritarians, and the assimilated minoritarians will have an acceptable, conformist behavior, the majoritarian community will grow, and they will preserve their subjective dualities. The minoritarians will change their habits and customs, but they will gain the wished assimilation, which means a subjective benefit, on the other hand they can gain some objective benefits too (ex. higher wages).

The above mentioned game has two Nash-equilibria points, which are the N, R and the C, A states, if we presume that the partners are not knowing what the behavior of the other partner will be, they choose the N, R state. In this case the decisions can be altered, if one of the partners is trying to convince the other for a longer period about his good willing, with the help of a favorable behavior. In this way the equilibria can move to the C, A state. If there is no positive response to their initiative, however the chance of the turning to a non-favorable behavior will increase, despite of the previous favorable changes. This process can be shown by the modification of the pay-off matrix. The continuous, unfavorable policy from the point of view of the majoritarians will decrease the subjective benefits of the accepting behavior. This decrease can happen in a very short run, so it seems that it could be a good decision for the minoritarians to produce the positive response to the favorable policy in time, because this policy can turn in its opposite in very short time. The modified game can be described by the following pay-off matrix:

Majoritarian	**Reject** / R	**Assimilate** / A
Minoritarian		
Nonconform / N	1, 1	3, -4
Conform / C	-1, 3	5, 2

Tabl.2.

In this game the dominant strategy for the majoritarians is the rejection, and the equilibria point of the game is the N, R state.

B type relation.

Majoritarian	**anti-Minoritarian** / M	**Pro-minoritarian** / P
Minoritarian		
Reject / R	0, 0	5,-1
Assimilate / A	-1, 5	1, 1

Tabl. 3.

The meaning of the values of the pay-off matrix, if we make abstraction of the actual values, is:

R, M: The majoritarian policy is anti-minoritarian, and the minoritarians don't want to assimilate: neither of the partners is gaining anything (presuming that the anti-minoritarian policy doesn't mean the extermination of the minoritarians). On the one hand the majoritarian legislation is containing laws which forces the assimilation of the minoritarians without any success, on the other hand the minoritarians are preserving their separation, but they receive objectively negative discrimination.

R, P: The majoritarian policy is pro-minoritarian, but the minoritarians don't want to assimilate, the minoritarians will gain the benefits of the situation, and the majoritarians will loose a lot. The majoritarians are canceling the anti-minoritarian legislation, possibly they offer some positive discrimination, but they don't gain anything in exchange. The minoritarians are using the possibilities of the pro-minoritarian legislation, without giving up their separation.

A, M: The majoritarian policy is anti-minoritarian, and the minoritarians try to go ahead on the way of the assimilation, the majoritarians will gain the benefits, and the minoritarians will loose a lot. The majoritarians are making anti-minoritarian laws to fasten the assimilation process, and they will gain substantial benefits. The minoritarian are accepting some of the assimilation claims of the majority, and giving up their positions they will loose subjectively, without having any subjective or objective benefit.

A, P: The majoritarian policy is pro-minoritarian, and the minoritarians are ready to make some steps on the way of the assimilation, both of the partners will gain something. On the one hand the partial readiness for the assimilation, of the minoritarians is a positive event for the majoritarians, on the other hand they are giving some concessions, which are interpreted subjectively as being negative. On the one hand the minoritarians are giving up some of their separationist claims, which mean losses for them, on the other hand the positive changes in the legislation mean to satisfy some of their claims, which have positive consequences for them. Finally the decreasing of the tensions can mean an objective benefit for both partners (ex. the increasing of the foreign investments).

The dominant strategy of this game is the **R, M** state, which means the rigid rejection of the idea of the assimilation, and the strong anti - minoritarian legislation. The moving of the dominant strategy to the **A, P** state needs the modification of the pay-off matrix by some adequate policy.

Modifying the Pay-off Matrix

Here I'll present a very simplified real example, after which the possibility of application of the presented method will be studied in some other cases.

The example is the reform of American educational policy in the 50s ([HERRNSTEIN94], p447 - 477), when the federal governmental subventions were available only for schools which lacked racial discrimination. If a school had racial restrictions, e.g. no black pupils were admitted, it was noneligible for a federal subvention, and the federal subvention increased with the increase of the proportion of the black pupils.

The modification of the educational policy meant, that if a school behaved as a rejectionist, it was not given any subvention, and it remained only with its subjective benefit of having only white pupils. In the case when the school behaved in an assimilationist way, accepting the black pupils, it gained the federal subventions and the black pupils gained a better education than before. The situation is a modified *A* type situation. The pay-off matrix of the new game is the following.

Majoritarian / **Minoritarian**	**Reject** / R	**Assimilate** / A
Nonconform / N	1, 0	4, 2
Conform / C	-1, 0	7, 7

Tabl. 4.

In this case the dominant strategy of the game is the **C, A** state. So by this policy the modification of the pay-off matrix was possible in such a way that both partners got the best choice for themselves, independently of their distrust and counter-feelings.

Of course, the schools took into consideration the previously existent customs too, and accepted only the minimal sufficient number of the black pupils, otherwise too many white pupils would have gone to whiter schools and the school would have lost its reputation. However, in the long run this modification changed the customs too, and generally it made acceptable the coeducation of the black and white pupils. This secondary, long run effect increases the attractiveness of the **C, A** strategy.

Assuming a similar policy modification in the case of a *B* type relation, the result will be the adverse. In this case the modified pay-off matrix will be:

Majoritarian / **Minoritarian**	**anti-Minoritarian** / M	**Pro-minoritarian** / P
Reject / R	0,-5	10, 4
Assimilate / A	-1, 0	6, 6

Tabl. 5.

The new policy is sanctioning the anti-minoritarian behavior, which is observable through the decreasing of the pay-offs in the column **M**. In this latter case (anti-minoritarian behavior) the minoritarians will not use the benefits either, so their pay-offs will remain unchanged in this column. The pro-minoritarian behavior is rewarded, which causes the increase of the pay-offs for both partners in the column **P**.

The result of the new policy is that the pro-minoritarian behavior will be the dominant strategy for the majoritarians, and the equilibria will move to the state **R, P.** This modification cannot be sustained in the long run, because it will increase the value of the subjective losses of the majority, which will also lead to the modification of the pro-minoritarian policy. In the case when an external force doesn't permit the modification of the pro-minoritarian policy, the subjective value of the anti-minoritarian policy will increase, which modifies the pay-off matrix in the following way:

Majoritarian / **Minoritarian**	**anti-Minoritarian/** M	**Pro-minoritarian** / P
Reject / R	0, -1	10, -2
Assimilate / A	-1, 1	6, 1

Tabl. 6.

The equilibrium point of the new game returns to the point **R, M,** that is to the equilibrium point of the anti-minoritarian behavior and so to the rigid rejection of any form of assimilation.

To successfully modify the previously presented case, it is necessary to change the policy in such a way which can make attractive the idea of giving up the rigid separation for the minoritarians.

Applying the policy of the beginning example, in an *A* type case, when there is no objective discriminating factor, or it exists only in a very low proportion, the discriminative subvention policy will influence negatively the subjective pay-offs of the minoritarians. In this case the pay-off matrix will be the following:

Majoritarian / **Minoritarian**	**Reject** / R	**Assimilate** / A
Nonconform / N	1, 0	4, 2
Conform / C	-1, 0	3, 7

Tabl. 7.

In this game the dominant strategy for the minoritarians is the nonconformist behavior, and the game equilibrium will be in the point **N, A.** The result in the long run is the same as in the case of the *B* type relation.

So, this kind of policy, will be successful in an *A* type situation, when there is an objective discriminating factor, and which is accepted without problems by the minoritarians.

Beyond the pay-off matrix

In the following part the analysis will show how the values of the pay-off matrix can be approximated. The economical model used here is simplified too, but it is based on the real social background of the interethnic relations presented in the theoretical model (similar treatment of the sociological models can be found in [BECKER94]).

In order to build up the model, let us suppose that the country, where the two ethnic communities live, receives a foreign investment aid, and the received money will be split by the governmental policy between the settlements of the two communities. Let us say F is the total sum of the aid, x_1 is a part of it which is invested in majoritarian settlements, and x_2 the part which is invested in minoritarian settlements. So, we can write the equation of the budget limit, which is:

$x_1 + x_2 = F.$

Furthermore we will analyze, which combinations of the x_1 and x_2 values will lead to the same popularity of the government. Those points will form the iso-popularity curves. The shape of those curves will be similar to the followings :

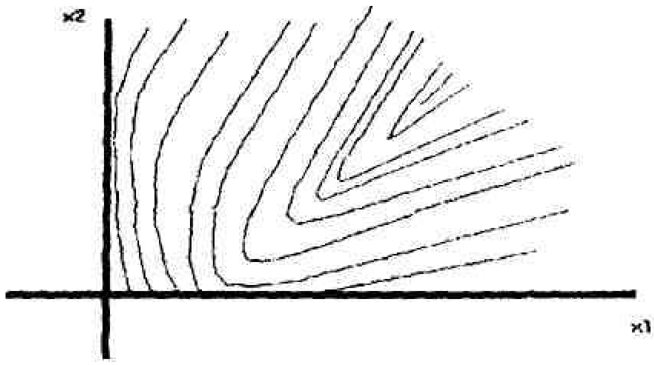

Fig. 1.
Izo-popularity curves

As it can be seen in the above figure, the curves with low popularity cross the x_t axis, which means that the low popularity levels can be achieved by the government, even if the minoritarians will receive nothing from the investment aid. It can be also seen that the highest izo-popularity curve approximates a half-straight line, the slope of which depends on the ratio between the number of the members of the two communities (each settlement will receive a part of the aid, in accordance with the number of its inhabitants). At the same time it can be observed that such a distribution is possible only if $x_1+x_2 = F_0$, which means that the total sum must be sufficiently large, which generally is not the case (in the model we assume that F is much smaller than F_0).

According to our simplified model, the government will choose that kind of investment policy which gives them the highest popularity, which means that the line of the equation $x_1 + x_2 = F$ is tangent to an izo-popularity curve.

Assuming that F is sufficiently small (in practice this is the general case), which means that the important x_1 and x_2 values are sufficiently small as well, the izo-popularity curves can be approximated by the following curves:

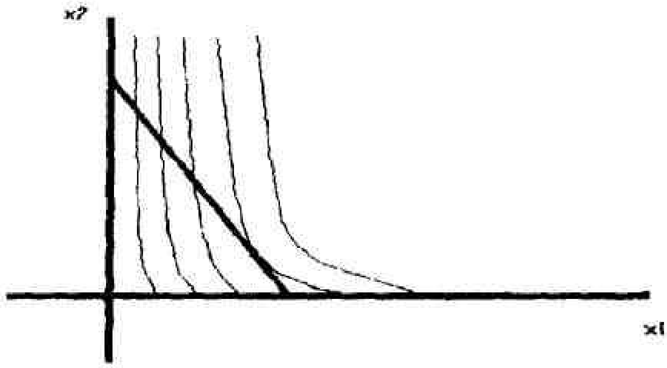

Fig.2.

Approximation of the izo-popularity curves, when F is small

Let $V(x_1, x_2)$ be the function which gives the value of the popularity for a combination of x_1 and x_2. So, the equation of the izo-popularity curves can be written in the form $V(x_1, x_2) = k$. The izo-popularity curves from the Fig 2. can be given in the form of $V(x_1, x_2) = a_1x_1 - a_2 + v(x_2) = k$ (V is a cuasi-linear two variable function), where $v(x_2)$ is an increasing function of x_2, which has a positive limit when x_2 is increasing to infinity, and its value for $x_2 = 0$ is negative. The a_1 and a_2 constants characterize the majoritarians, and their values are positive. The lower values of $v(x_2)$ means the higher sensitivity of the minoritarians, and the higher values of a, means that the majoritarians are more anti-minoritarians ($v(x_2)$ can be written in the form of $v(x_2)=-b_1 / (x_2 + m) + b_2$, where b_1, b_2 and m are positive constants).

How this model can be used for the calculation of the values of the pay-off matrix?

In fact the behavior of the partners will modify the shape and / or the position of the izo-popularity curves. So, if we can find out how those curves will be modified by a certain behavior pair, we will be able to find out the new distribution policy too. Comparing this with the previous distribution policy the result will be the modification of the sums received by the partners, which indicate the objective gain or loss of them. The subjective gain or loss can be calculated on the base of the difference between the partial popularity value of the previous and the actual izo-popularity curves, for each of the partners (the partial popularity value is the part of the total popularity value of the izo-popularity curve, which is resulted from the popularity of the government within one of the communities).

We will carry out this analysis in the case of a *B* type relation. First we will solve the general problem of the resource allocation of the government, taking into account its popularity and the available resources. Let the popularity function be:

$$V(x_1, x_2) = a_1x_1 - a_2 + -b_1 / (x_2 + m) + b_2.$$

In this case the value of x_2 as a function of x_1 and k, is:

$$x_2 = g(x_1) = -m + b_1 / (a_1x_1 - a_2 + b_2 - k)$$

Writing the condition of the tangentiality we will find out the x_1 (the value of the derivative of $g(x_1)$ must be -1, because the derivative of the x_2= F - x_1, is -1 everywhere):

$$g'(x_1) = -b_1a_1 / (a_1x_1 + b_2 - a_2 - k)^2 = -1.$$

Determining x_1 and x_2 we will get:

$$x_1 = (\sqrt{a_1b_1} + a_2 + k - b_2) / a_1, \text{ respectively } x_2 = \sqrt{(b_1 / a_1)} - m.$$

Considering the equation of the budget limit, we can determine the value of x_1 and k too, as a function of the parameters:

$$x_1 = F + m - \sqrt{(b_1 / a_1)}, \text{ respectively } k = a_1(F + m) + b_2 - a_2 - 2\sqrt{a_1b_1}.$$

Based on the above results it can be easily observed that the values of x_1 and x_2 depend only on a_1 and b_1, and that the modification of the parameters a_2 and b_2 modifies only the value of k.

Let the following situation be the case of anti-minoritarian behavior and rigid separatism.

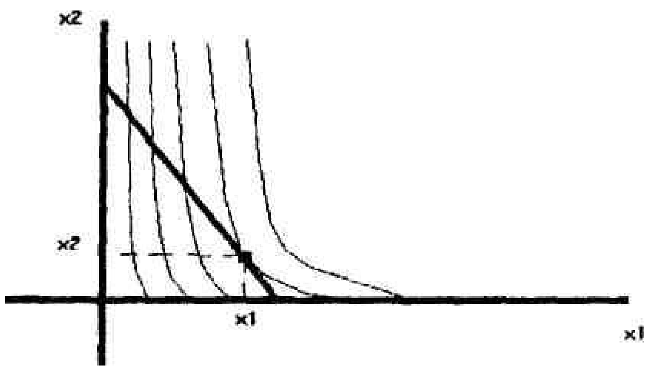

Fig. 3.
Anti - minoritarian policy / Rigid separationism

Let suppose that in this situation the gain of the both partners is 0.

If the majoritarians do not modify their behavior, but the minoritarians give up some of their separationist claims, that means that there will be less discontent because of the nonfavourable behavior of the majority (the value of b_1 decreasing ($\underline{b}_1 < b_1$) and the result will be the increasing of the slope of the izo-popularity curves, because the values of the new $\underline{v}(x_2)$ function will be less negative for small values of x_2. The new situation is shown in the following figure:

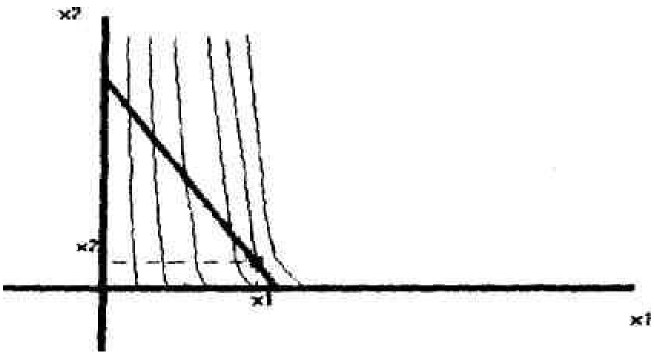

Fig. 4.
Anti - minoritarian policy / Less separationism

It can be seen on the above figure, that the minoritarians will suffer an objective loss, and majoritarians will objectively gain. The subjective loss or gain of the minoritarians depends on the $v(x_2)$ and $\underline{v}(x_2)$ functions, while the majoritarians will gain subjectively too. The new popularity level will be higher than the previous one.

If the majoritarians change their behavior to be pro-minoritarian, without any modification of the minoritarian's behavior, we will have the new value of the constant $\underline{a}_1$ smaller than the value of the previous a_1. The new situation is.

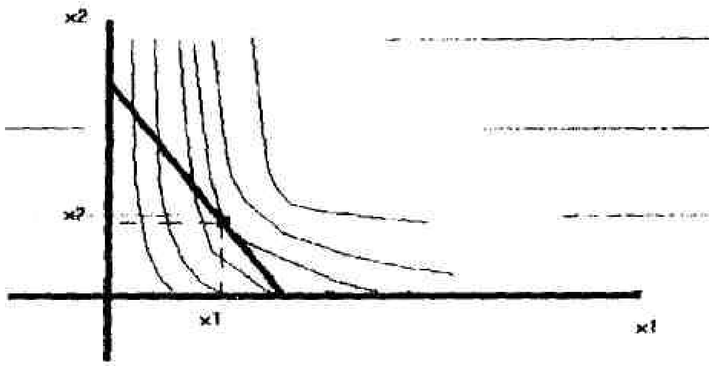

Fig. 5. Pro - minoritarian policy / Rigid separationism

It is obvious that the minoritarians will gain from the change, while the majoritarians will loose. The change means a subjective loss for the majoritarians, and a subjective gain for the minoritarians. The new popularity value depends on the parameters of the popularity function.

If both partners change their behavior positively, the result can be interpreted in the language of the model as $\underline{v}(x_2) < v(x_2)$ and $\underline{a}_1 < a_1$.

The new situation is shown in the following figure:

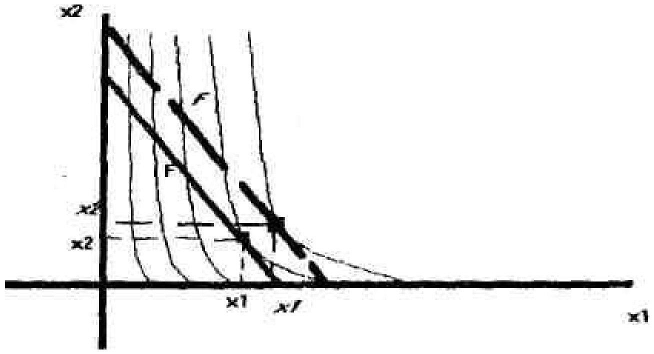

Fig. 6.
Pro-minoritarian policy / Less separationism

The objective gain or loss of the partners in the new situation depends on the parameters of the popularity function. The minoritarians will gain subjectively, because the new distribution policy will be near the previous one, and this point has now a higher partial popularity level from the point of view of the minoritarians. In the case of the majoritarians the situation is not so clear. Their partial popularity level will decrease for every x_1, which can be interpreted as a subjective loss, or as the decreasing importance of the aid distribution in the determination of the popularity of the government from the point of view of the majoritarians (of course, a more detailed model can show a better picture). Furthermore the new situation is more stable from the point of view of the aid giving partner, which can cause the increase of the value of F (F > F), which will mean for both partners a subjective and an objective gain as well.

The next question is how a policy can be carried out in order to lead to a situation which is favorable for both partners. This means in the language of the pay-off matrix, that the modification of the values of the matrix should be made in such way that the dominant strategy will be the one, which is individually the most favorable for both partners.

The effects of different policies

Within the presented model the different policies mean the modification of the shape or position of the izo-popularity curves, or the modification of the budget limit curve.

The government, having little influence on the behavior of the minoritarians, can modify the values of the a_1 and a_2 constants, in order to modify the shape or the position of the izo-popularity curves. As we have seen before such a policy has no long run effect without the adequate minoritarian response, which is not very probable.

Another possibility is the modification of the budget limit curve, which can also result as an effect of the conditions imposed by the aid giving external partner.

One possibility is to fix the minimum ratio or the minimum sum of the aid part given to minoritarian settlements. For these cases the situation is shown in the following figures:

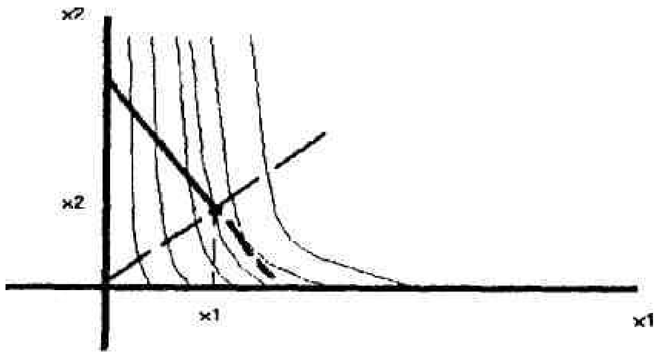

Fig. 7.
Modifying the budget limit curve by minimum ratio

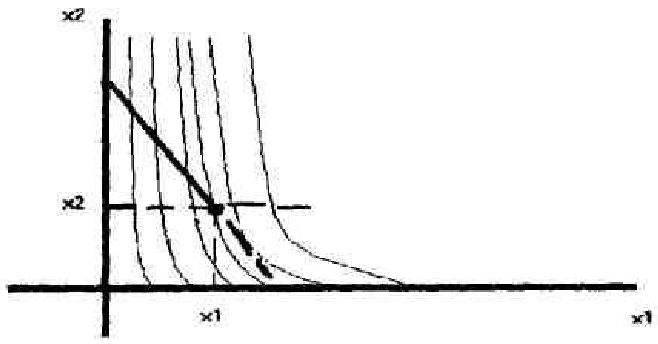

Fig. 8.
Modifying the budget limit curve by minimum amount

As we can see, in both cases, this modification means, that we don't take into consideration a part of the budget limit curve.

If the initial distribution was less favorable for minoritarians than the fixed minimum ratio or amount, its introduction will lead exactly to the distribution corresponding to the fixed ratio or amount, that is a corner-solution. In other cases the modification has no effect on the distribution policy. If the result will be a modification in the distribution policy, the new popularity level will be lower than in the case without any modification.

Furthermore, this modification damages the pay-off matrix. The possible modifications will not modify the shape of the izo-popularity curves, and the solution will remain a corner-solution.

The short run effect will be the improvement of the situation of the minoritarians, but in the long run this will lead to the decreasing of the stability, which can cause the change of the government and the formal or practical rejection of the aid (the conditions are not satisfied). Both of the long run effects are unfavorable for both communities.

Another possibility is to establish sanctions based on the ratio of the aid amounts received by the two communities. Mathematically the condition can be formulated in the following way:

$$x_1 + x_2 = F - t\, x_1 / x_2, \text{ where t is a penalization constant.}$$

The condition means that, if the ratio of x_1 / x_2 is greater, the total sum is lower. In this case we get x_1 as a function of x_2 in the following form:

$$x_1 = h(x_2) = - x_2 + F - t((F + t) / (x_2 + t) - 1).$$

Calculating the first and second order derivative of $h(x_1)$ we get:

$$h'(x_2) = -1 + t(F + t) / (x_2 + t)^2, \text{ respectively}$$

$$h''(x_2) = -2t(F + t)/(x_2 + t)^3$$

Based on the above results it can be seen that $h(x_2)$ is concave and it has a maximum point in the (0, F) interval. The situation is shown in the following Figure:

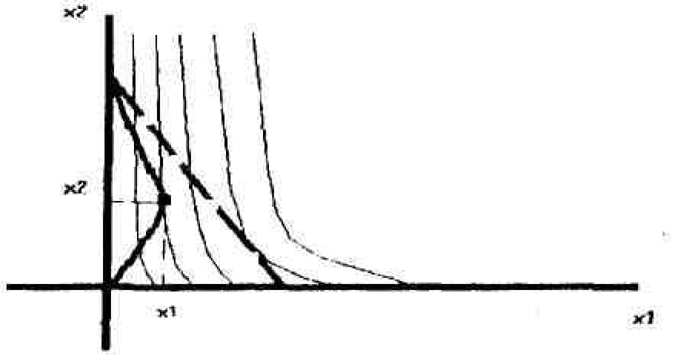

Fig. 9.
Modifying the budget limit curve by penalization

Based on the figure it is obvious that the result of the introduction of the penalization will be more favorable for the minoritarians than the previous situation. It is also observable, that after the introduction of the penalization the new popularity level will be lower than the previous.

If such a policy can be sustained for a sufficiently long period it can have as a secondary effect the modification of the shape of the izo-popularity curves, which also means the modification of the pay-off matrix. However the fact of penalization can have some negative effects, which can turn things toward the opposite direction.

Finally we will analyze the case, in which it is introduced a compensational amount based on the ratio of the aid amounts given to the two communities. So, if the ratio is favorable for the minoritarians, the whole sum of the aid will be increased.

Mathematically this case can be written as:

$x_1 + x_2 - F + t\, x_2 / x_1$, where t is a compensating constant.

We can find out x_2 from the above equation, as:

$$x_2 = f(x_1) = -x_1 + F - t + t(F-t)/(x_1-t).$$

For the first and second order derivatives of the function $f(x_1)$ we get:

$$f'(x_1) = -1 - t(F-t)/(x_1-t)^2,$$
$$f''(x_1) = 2t(F-t)/(x_1-t)^3.$$

It is obvious from the equations, that if x_1 takes values close to t, the values of x_2 will increase towards the infinite. So we need an additional constraining condition, which can be written as:

$$x_1 + x_2 <= F_0.$$

If t > F the equation f '(x_1) = 0 has solution, and the values of x_1 for these are:

$$x_{11} = t + 1 / \sqrt{t}(t - F), \text{ and } x_{12} = t - 1 / \sqrt{t}(t - F).$$

In this case the f "(x_1) is positive if x_1 < t, and negative if x_1 > t, so the graph of the function is convex, respectively concave on these intervals. Furthermore if x_1 is going to plus / minus infinite, then f(x_1) tends to the graph of the function x_2 = - x_1 + F. It is easy to observe that, if x_1 is tending to t from below f(x_1) is tending to plus infinite, and if x_1 is tending to t from above, f(x_1) is tending to minus infinite.

After these we can draw the graph of the function f(x_1). and it is possible to analyze the position of tangency point between the budget limit curve and the izo-popularity curves.

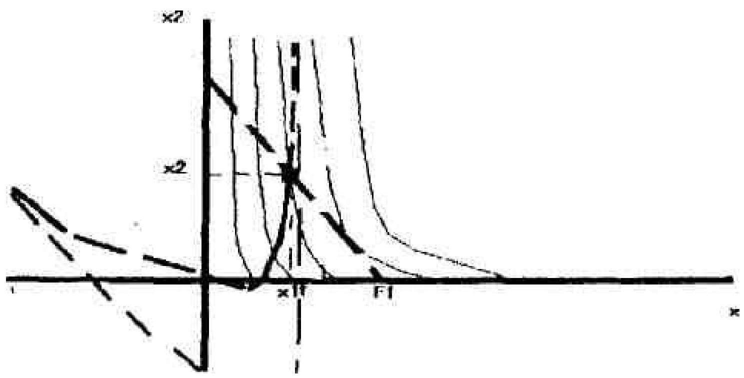

Fig. 10.
Modification of the budget limit curve with compensation, when t > F

It is easy to observe that we have a corner-solution, which is the same as the one obtained, that is as we would have the total sum equal to F_0 and a fixed ratio constraint. So the results will be the same as the results of the previously presented case. The single difference is that in this case F_0 is greater than F. On the other hand it is certain that the F_0 sum will be distributed, so we will have the same objective result as if we gave as an initially distributable sum. F_0, imposing a fixed minimum ratio. Some difference can result from the fact, that in the actual case the increase from F to F_0 is a reward.

The second case is when F > t. In this case the equation f '(x_1) = 0 has no solution. The convexity of the graph of the function will be exactly the adverse of the one in the previous case, so the graph is concave when $x_1 < t$, and convex when $x_1 > t$ Similarly the values of the function, near t will move in the adverse direction, than in the previous case In the infinity the function has the same behavior than in the previous case. So, the graph is:

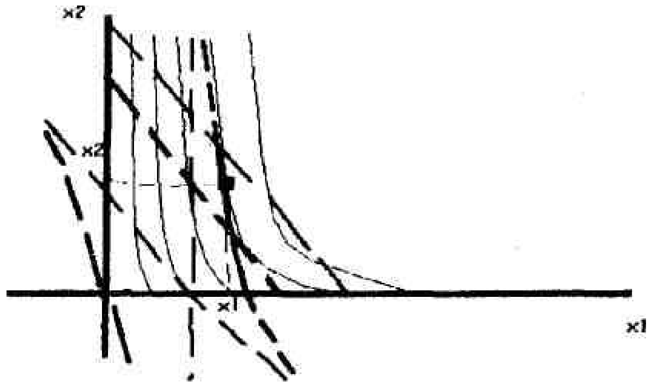

Fig. 11.
Modification of the budget limit curve with compensation, when F > t

Because F - t > 0 we have that

$$f'(x_1)<-1, \text{ for all } x_1.$$

Furthermore, because the slope of the izo-popularity curves is decreasing, as x_1 is increasing, we get that the tangency point of the modified budget line and the izo-popularity curve is situated higher (according to the x_2 axis), than the tangency point between this izo-popularity curve, and an unmodified budget line, which is tangent to the latter. This means that a popularity level can be obtained with a better distribution for the minoritarians, with this type of constraint, than without it.

More advanced treatment of the policy analysis can be made using dynamic optimization, by which is possible to incorporate in the model the time-dependent variations of the components of the model ([CHIANG92], p.98 - 130., p.240 - 264., [HOLLY89], p. 197 - 225).

Conclusions

The presented method of analysis might seem to be very theoretical and distant from the realities. However, when based on real data, it is possible to build up the graphs and functions of the model. In this way the method will be applicable to real situations

The method *could* be detailed with the enrichment of the used theoretical base, taking into account the effects of mere factor. From this point of view is very important to make investigations by methods of the other social sciences, to find out other important factors.

By this method it is possible to *analyze* the policies and their effects in a more objective way.

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